Active filter has stable notch, and response can be regulated

High Q of zeroes in transfer function is independent of component balance; notch depth depends on high gain, rather than precision of parts, and circuit-performance sensitivity to passive elements is low

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☐ Many tone-signalling systems require elimination or rejection of a single frequency or a narrow range of frequencies. To produce this stop band, transfer functions with a notch response have been achieved by both active and passive networks. However, all the circuits that have been used in the past have required accurately matched component values to produce deep notches. Unfortunately, aging and temperature variations can affect the capacitances or resistances of carefully matched components differently, with the result that

their match is degraded. Aging can thus reduce a rejection ratio of, say, 60 decibels to as little as 10 dB.

Now this need for perfect matching has been eliminated by an active filter in which the Q, or sharpness, of the null is a function of amplifier gain, rather than of precision balancing of passive components. The notch depth in the new network is constant so long as the gain remains high, even if resistors and capacitors drift. The active-filter network can also generate low-pass or high-pass filter blocks for frequencies above or below the rejection frequency. The passive component sensitivities of the network are 0.5 or less.

Considering null networks

Among the passive bandstop networks that depend on the precision with which components are matched is the symmetrical parallel-T network shown in Fig. 1(a). One condition for balance is that the ratio of the series and shunt capacitors must be proportional to the ratio of the series and shunt resistors ($C_2/C_1 = 4R_1/R_2$). This balance, which is independent of frequency, implies that the depth of the notch at the rejection frequency depends solely on the accuracy of passive component matching. To get 60 dB of rejection at the notch frequency, the ratio of C_1 to C_2 must be held within 0.1% over the temperature range of interest.

$$\omega_N = 1/2R_1C_1$$

(a)

$$\omega_N = \sqrt{(1/R_5C_3C_4)(1/R_1 + 1/R_2)}$$

(b)
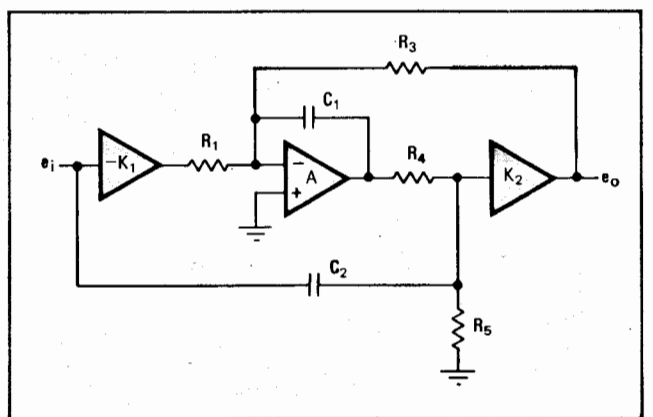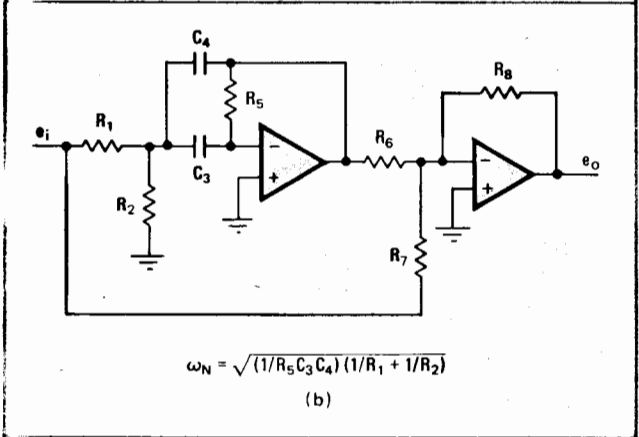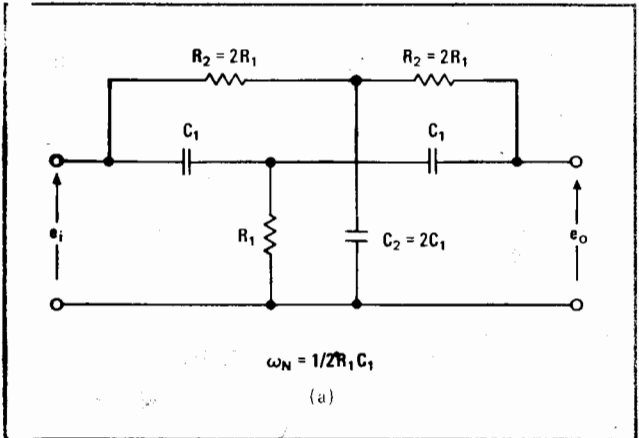
1. Null networks. Conventional notch filters may be passive circuits such as the bridged-T network in (a) or active circuits such as the subtractive arrangement in (b). In either type, the amount of signal rejection at the notch frequency depends upon ratios of passive components; therefore, component drift degrades performance.

2. Zero generation. New circuit can generate true zeroes at any frequency for which op amp has high gain. Notch depth is function of this gain alone, not of component ratios. Filter can combine low-pass or high-pass characteristics with notch. Pole and zero frequencies and pole Q-factor are independently adjustable.
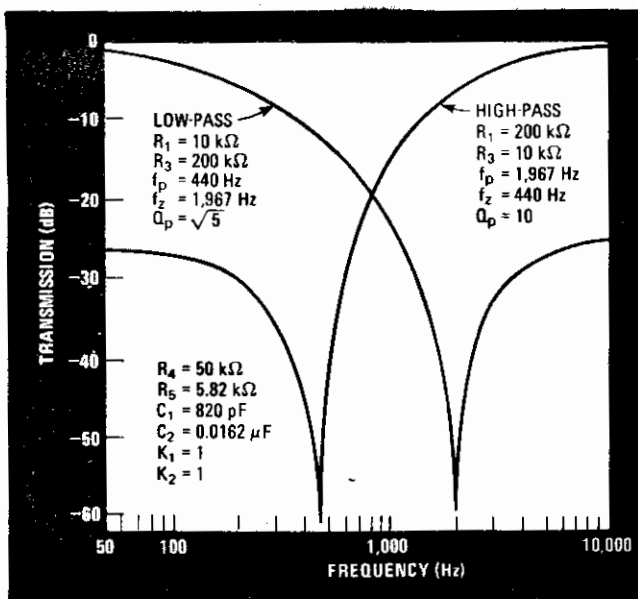
3. Characteristics. Interchanging R_1 and R_3 changes filter response from low-pass to high-pass. If R_1 is equal to R_3, filter has symmetrical notch characteristic. Quality factor Q_p, which is measure of how fast the response returns from the notch to its passband characteristic, depends on values of resistances and capacitances.
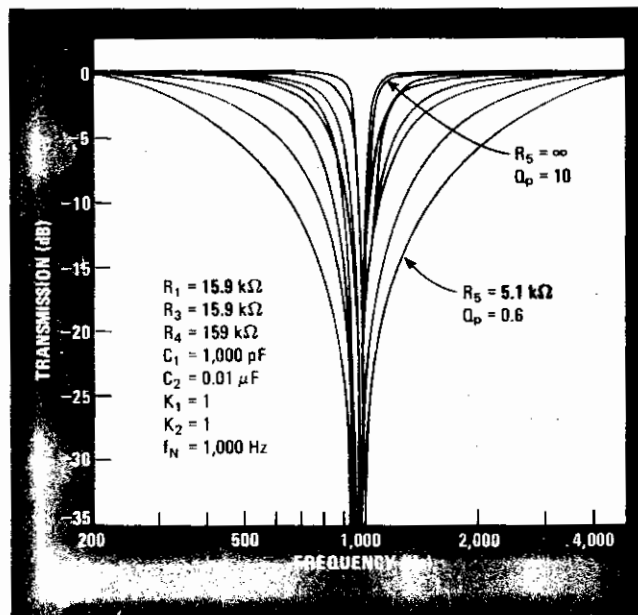
4. Adjustable. Response of unity-gain notch filter is varied by varying R_5 to change Q_p value. Notch frequency is 1 kHz.

Other null networks have active circuits that subtract one signal from another to produce a notch at the desired rejection frequency. As an example, Fig. 1(b) shows a multiple-feedback bandpass active filter connected to a summing amplifier. At the node of R_6 and R_7, the input signal is subtracted from the output of the filter section. The final transfer function is

$$\frac{e_o}{e_i}(s) = -\frac{R_8}{R_7}\left[\frac{s^2 + s(1/C_3R_5 + 1/C_4R_5 - R_7/C_4R_1R_6) + \omega_N^2}{s^2 + s\omega_N/Q_p + \omega_N^2}\right]$$

where ω_N is 2π times the notch frequency, and Q_p determines notch width. To produce a deep notch with this

circuit, the middle term in the numerator of the equation must be zero. That is,

$$R_5/R_1 = (R_6/R_7)(1 + C_4/C_3)$$

This expression shows that the amount of rejection at the notch frequency depends upon three ratios of passive components. Therefore, to maintain good notch depth, these ratios must be accurately set and maintained over the range of operating temperature.

Other bandpass filters, such as the state variable or biquad, may also be used; but they also require balancing of components, because the filter sections do not inherently generate transfer zeroes.

Generating transfer zeroes

The general form for the transfer function of an active filter is

$$\frac{e_o}{e_i}(s) = \frac{s^2 + (\omega_z/Q_z)s + \omega_z^2}{s^2 + (\omega_p/Q_p)s + \omega_p^2}$$

where ω_z and ω_p are the radian frequencies for the zeroes and poles, and Q_z and Q_p are the corresponding quality factors. For infinite Q_z, the coefficient of s in the numerator would be zero; this fact suggests that a circuit with a large gain factor in the denominator of this s-coefficient must have high Q_z. Such a circuit is shown in Fig. 2. The coefficient of s in the numerator of its transfer function is

$$\omega_z/Q_z = (1/R_1 + 1/R_3)/C_1(1+A) \qquad (1)$$

where ω_z^2 is the constant term in the numerator, $AK_1/R_1R_4C_1C_2(1+A)$. The value of Q_z is therefore given by the expression

$$Q_z = [K_1C_1A(A+1)/R_1R_4C_2]^{1/2}R_1R_3/(R_1+R_3) \quad (2)$$

If the gain of the operational amplifier, A, is large (on the order of 10^4), Q_z is greater than 200. For such high values of A, the transfer function of the circuit in Fig. 2 is effectively

$$\frac{e_o}{e_i}(s) = K_2\frac{s^2 + [K_1/(R_1R_4C_1C_2)]}{s^2 + [s(R_4+R_5)/(C_2R_4R_5)] + K_2/(R_3R_4C_1C_2)} \quad (3)$$

Since this equation has no s term in the numerator, the transmission function has a deep notch at the frequency given by

$$\omega_z^2 = \frac{K_1}{R_1R_4C_1C_2} \qquad (4)$$

The notch frequency may shift if component values drift, but the depth of the notch will not be materially affected by such drift.

Calculating circuit performance

Equation (3) shows that the zero and pole frequencies, and Q_p, for the circuit in Fig. 2 are given by Eqs. (4) and

$$\omega_p^2 = \frac{K_2}{R_3R_4C_1C_2} \qquad (5)$$

$$Q_p = \left[\frac{K_2C_2}{R_3R_4C_1}\right]^{1/2}\frac{R_4R_5}{R_4+R_5} \qquad (6)$$

$$\left[\frac{\omega_z}{\omega_p}\right]^2 = \frac{K_1}{K_2}\frac{R_3}{R_1} \qquad (7)$$

The gain of the circuit in Fig. 2 at zero frequency is

$$\frac{e_o}{e_i} = \frac{K_1 R_3}{R_1} \qquad (8)$$

and at infinite frequency is

$$\frac{e_o}{e_i} = K_2 \qquad (9)$$

Thus, K_1, K_2 and R_3/R_1 can be used to set the transfer gain below and above the zero frequency ω_z.

Equations (7), (8), and (9) show how to select component values so that the circuit will function as a low-pass, high-pass, or notch filter: if $K_1 = K_2 = 1$,
■ R_3 greater than R_1 gives a low-pass filter
■ R_3 equal to R_1 gives a notch filter
■ R_3 less than R_1 gives a high-pass filter
Figure 3 shows the result of interchanging R_1 and R_3 to convert the filter section from low-pass to high-pass.

Resistor R_5 can be used to adjust Q_p without affecting the zero or pole frequencies; in fact, the circuit can be designed without R_5. If R_5 is omitted, Q_p is

$$Q_p\Big|_{R_5 = \infty} = \left[\frac{K_2 R_4 C_2}{R_3 C_1}\right]^{1/2} \qquad (10)$$

Resistor R_5 can then be added to lower the total Q_p if the application requires that the Q_p be adjustable; to get a given value of Q_p, R_5 should be

$$R_5 = \frac{1}{(1/Q_p)(K_2 C_2/R_3 R_4 C_1)^{1/2} - 1/R_4} \qquad (11)$$

When R_5 is included in the circuit its value should be of similar to that of R_4; otherwise, the output of the op amp may saturate at the notch frequency. It is good practice to let $R_5 = R_4$ when using R_5 in the design. This value of R_5 results in low sensitivity of Q_p to R_4, as shown below.

The source that drives one of these filter sections should have a low resistance. The source resistance has no effect on the notch frequency, but it does affect the overall gain, ω_p, and Q_p.

The equations defining the component values may be further simplified for notch filters, in which $R_1 = R_3$ and therefore $\omega_z = \omega_p = \omega_N$, where $\omega_N^2 = 1/R_1 R_4 C_1 C_2$ is the square of the notch frequency. Two sets of notch-filter equations that are useful to the designers are given in Table 1. One set of equations applies for the case where R_4 is equal to R_5, and the other set applies when R_5 is infinite; $K_1 = K_2 = 1$ is assumed throughout.

Comparison of Eqs. 14 and 17 shows that the component value spread is reduced by a factor of four if R_5 is set to infinity; but the flexibility in adjusting Q_p is lost.

To design a filter for a given ω_N, an appropriate Q_p is chosen, and C_1 is made equal to C_2 at some fraction-of-a-microfarad value that yields convenient resistor sizes.

Figure 4 shows the response of a unity-gain notch section where f_N is 1,000 Hz and Q_p is varied over the range from 0.6 to 10 by varying R_5.

In applications where it is required to notch out a significant bandwidth, as in the band-rejection filter for a
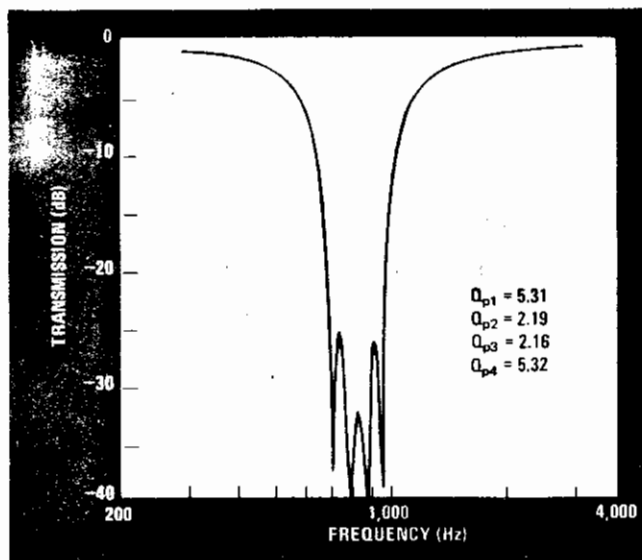
5. Stopband. Four cascaded notch sections with different rejection frequencies and different Q_p values produce a bandstop filter.

$Q_{p1} = 5.31$
$Q_{p2} = 2.19$
$Q_{p3} = 2.16$
$Q_{p4} = 5.32$

**TABLE 1
NOTCH-FILTER EQUATIONS
(FOR $K_1 = K_2 = 1$)**

FOR $R_4 = R_5$		FOR $R_5 = \infty$	
$R_1 = R_3 = \dfrac{1}{2\omega_N Q_p C_1}$	(12)	$R_1 = R_3 = \dfrac{1}{\omega_N Q_p C_1}$	(15)
$R_4 = R_5 = \dfrac{2Q_p}{\omega_N C_2}$	(13)	$R_4 = \dfrac{Q_p}{\omega_N C_2}$	(16)
$\dfrac{C_2}{C_1} = 4 Q_p^2 \dfrac{R_1}{R_4}$	(14)	$\dfrac{C_2}{C_1} = Q_p^2 \dfrac{R_1}{R_4}$	(17)

**TABLE 2
COMPONENT SENSITIVITIES (FOR CIRCUIT OF FIG. 2)**

	R_1	R_3	R_4	R_5	C_1	C_2	K_1	K_2
ω_z	$-1/2$		$-1/2$		$-1/2$	$-1/2$	$1/2$	
ω_p		$-1/2$	$-1/2$		$-1/2$	$-1/2$		$1/2$
$\begin{matrix}Q_p\\R_5 < \infty\end{matrix}$		$-1/2$	$\dfrac{1 - R_4/R_5}{2(1 + R_5/R_4)}$	$\dfrac{1}{1 + R_5/R_4}$	$-1/2$	$1/2$		$1/2$
$\begin{matrix}Q_p\\R_5 = \infty\end{matrix}$		$-1/2$	$1/2$		$-1/2$	$1/2$		$1/2$

Touch-Tone telephone receiver, individual notch sections can be cascaded. Figure 5 shows the response of four cascaded notch sections for such an application; the notch frequencies are 697, 770, 862, and 941 Hz. The frequencies from 700 to 1,000 Hz are rejected by 25 dB. And since resistor R_4 is common to both ω_z and ω_p, the notch frequency is adjusted by trimming R_4.

The sensitivities of the singularities and of Q_p to fractional changes of passive-component values are shown in Table 2. For $R_5 = R_4$ or for R_5 equal infinity, all sensitivities are ½ or less, resulting in active filter sections that are stable with respect to component drift. □